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Simulation of Betatron X-Ray Radiation as an Indirect Measurement of Electron Trajectories in a Laser-Wakefield Accelerator

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Laser-wakefield accelerators (LWFA) only a few centimeters long have generated ultrarelativistic electrons and synchrotron-like x-rays. One barrier to using LWFA's for accelerator and x-ray source applications is the current inability to directly measure and characterize the betatron oscillations of electrons in the plasma. We have developed Mathematica/MATLAB simulations to model betatron oscillations and x-ray generation in a LWFA. By fitting simulation parameters to obtain closest agreement with experimentally measured betatron x-ray beam profiles, we obtain indirect measurements of experimental electron trajectories.

I. INTRODUCTION

Conventional particle accelerators and synchrotron facilities are limited to acceleration gradients of ~ 100 MeV/m. The laser-wakefield accelerator (LWFA) technique has demonstrated acceleration gradients $\sim 1000\times$ greater than conventional accelerator facilities, bringing forth the potential for meter-scale accelerator and synchrotron facilities. We have developed a code in both MATLAB and Mathematica (for wider user availability) to model the betatron motion and x-ray generation in a LWFA. We use experimental measurements to initially determine simulation parameters, then fit parameters such that the simulated betatron beam profile closely matches the experimental profile. The result provides key information about the electron trajectories (e.g. oscillation amplitude, trajectory length) in the wakefield cavity, thereby serving as an indirect measurement of the electron oscillations.

A. Electron Acceleration

The LWFA technique uses a high-intensity, fs-pulsed laser to drive a plasma wave on which electrons “surf” to rapidly gain energy. The leading edge of the laser ionizes atoms in a gas and the laser’s ponderomotive force (i.e. radiation pressure) expels the freed electrons, thereby producing a cavity of positive ions. This cavity, referred to as the wake, has a maximum “blowout” radius of $R_{max} \approx 2\sqrt{a_0}\omega_p$. Here, a_0 is the laser parameter and ω_p is the plasma frequency, given

by $a_0 = eA/mc$ and $\omega_p = \sqrt{n_e e^2 / \epsilon_0 m_e}$, respectively. In the bubble regime, the laser expels nearly all plasma electrons and travels within the front portion of a nearly spherical wake. The space charge force of the positive ions in the wake attracts freed electrons toward the back of the bubble, where they may be “injected.” Electrons then experience an accelerating field of ~ 10 - 100 GeV/m, which decays approximately linearly to zero at the center of the wake. The electrons accelerate over a dephasing length $L_{deph} \approx (P[TW])^{1/6} (10^{18} \text{ cm}^{-3} / n_e)^{4/3}$ before reaching the center of the wake, thus limiting their energy gain to $W_{max} [MeV] = E_z L_{deph} \approx 0.37 (P[TW])^{1/3} (n_e / 18^{18} \text{ cm}^{-3})^{-2/3}$. Here, E_z represents the electric field averaged over the wake structure. Since the field decays linearly, E_z can be approximated by $E_{max}/2$, where $E_{max} = \sqrt{a_0}$ is the maximum accelerating field at the back of the bubble [1].

In addition to the longitudinal accelerating field, electrons trapped slightly off-axis experience a transverse field. This causes them to oscillate, in what is referred to as “betatron” motion.

Combining the longitudinal and transverse accelerations, the equation of motion for an electron in a LWFA is given by

$$\frac{d\vec{p}}{dt} = -\frac{m_e \omega_p^2}{2} \hat{r} + e E_{max} (z - L_{deph}) \hat{z} \quad (1)$$

where \vec{p} is the electron’s momentum, \hat{r} is the unit vector locating the electron position relative to the center of the wake, and z is the longitudinal distance traveled by the electron since its initial injection into the bubble.

B. Betatron Radiation

The betatron oscillations cause electrons to emit electromagnetic radiation—primarily in the form of x-rays—in a low-divergence beam (<50 mrad at HWHM) with a duration of several femtoseconds. The betatron x-rays are collimated, somewhat coherent, and possess a source size of several μm (as verified by the simulations we have performed). The total radiated energy is proportional to γ^4 , so the highest energy electrons dominate the radiation profile. The spatial profile and frequency spectrum of the betatron x-ray beam is determined by the relation [2]

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \times \left| \int_{-\infty}^{\infty} \frac{\hat{n}}{(1 - \vec{\beta} \cdot \hat{n})^2} \exp \left[i\omega \left(t - \frac{\hat{n} \cdot \vec{r}(t)}{c} \right) \right] dt \right|^2. \quad (2)$$

Here, $\beta = v/c$ is the electron velocity normalized to c , \hat{n} the direction of observation, I the intensity of radiation, ω the frequency of radiation, and Ω the solid angle over which the electron radiates. The HWHM angular divergence of the betatron beam can be obtained through the relations $\theta \sim K/\gamma$ in the oscillation direction and $\theta \sim 1/\gamma$ in the transverse direction, where $K = (1.33 \times 10^{-10}) \sqrt{\gamma n_e [\text{cm}^{-3}]} r_\beta [\mu\text{m}]$ is the dimensionless wiggler strength. Typical values of K for our experiments were $K \approx 5 - 10$, so that the betatron motion and x-ray generation were best characterized by the wiggler regime ($K \gg 1$).

II. METHODS

A. Experimental

Experiments were performed at the Jupiter Laser Facility at Lawrence Livermore National Laboratory. The 200 TW, 60 fs Callisto laser was focused by a $f/8$ off-axis parabola to a focal spot of $10\text{-}12 \mu\text{m}$ (HWHM) into a $2\text{-}10 \text{ mm}$ gas cell, as shown in Figure 1. The cell was filled to pressures of $6 \times 10^{18} - 1 \times 10^{19} \text{ cm}^{-3}$ with pure He or He doped with small concentrations ($<10\%$) of N_2 . Upon exiting the gas cell, the ultrarelativistic electrons were deflected by a 0.42 T magnet and recorded by 2 photo-stimulated luminescent (PSL) image plates (IP's) at a distance of $z = 62.35 \text{ cm}$ from the exit of the gas cell. The “2 screen method” was used to determine the electron beam trajectory and consequently the electron energy

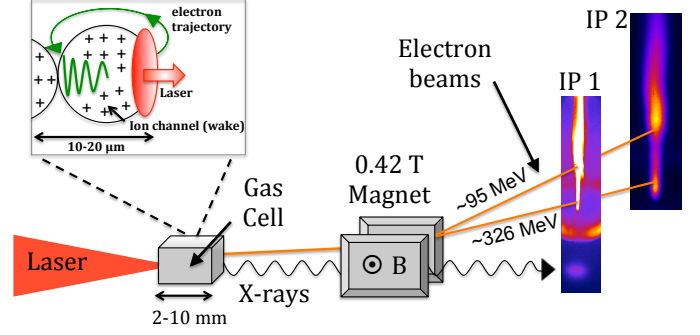


FIG 1 (color). Schematic of the experimental setup showing the 800 nm Callisto laser focusing into the gas cell. Electron trajectories for 2 different energies (labeled) are shown deflected onto the upper part of IP 1 and IP 2. Betatron x-rays pass undeflected through the magnet onto bottom of IP 1. (Inset) Diagram of electron trajectory in LWFA over $10\text{-}20 \mu\text{m}$ region of gas cell.

spectrum without ambiguity [3]. This measurement was used to determine the electron energies to be input to each simulation. In particular, it determined the maximum possible final electron energy and any prominent energies in the spectrum. The IP's simultaneously recorded the spatial profiles of the betatron x-rays. Using the relation $\theta \sim 1/\gamma$ in the transverse direction, we obtained the minimum possible final electron energy. Namely, the relation $\gamma_{\min} < 1/\min(\theta^{(h)}, \theta^{(v)})$ enforced a minimum on the electron's final energy [4].

B. Simulations

The simulations assumed an electron with initial relativistic factor of $\gamma_\phi = \frac{\omega_0}{\sqrt{3}\omega_p}$ [1], where ω_0 is the laser frequency. The electron trajectory was then calculated numerically using a 4th order Runge-Kutta 4th algorithm to solve Eq (1). From the trajectory, Eq (2) was solved numerically to obtain both the frequency spectrum and the spatial profile of the betatron radiation. For satisfactory results, the step sizes used to numerically evaluate Eq (2) were $\Delta\omega \leq 100 \text{ eV}$, $\Delta_{x,y} \leq 0.4 \text{ mrad}$ ($\sim 250 \mu\text{m}$ pixels at $z = 62.35 \text{ cm}$). The IP response, which is highly dependent on the incident photon energy, was taken into account in the simulations.

Input parameters were determined from experimental data. In particular, the simulations required both the gas density and laser energy used in the experiment. The electron injection radius $r_{0\beta}$ was initially approximated using the

K/γ relation in the oscillation direction, and was furthermore restricted to be less than R_{\max} . We determined the optimal value of $r_{0\beta}$ by iterating with slightly different radii until obtaining the closest agreement with the divergence of the experimental beam profile. Similarly, the final electron energy was iteratively adjusted to best fit the curvature and peak shape of the experimental beam profile. Depending on the electron energy spectrum and betatron profile, either a single electron or multiple (up to 16) electrons at various injection angles were used to simulate the beam profile.

III. RESULTS

The results for one such simulation are displayed in Figure 3. The laser energy and gas density were $E_{\text{Laser}} = 7.7$ J and $n_e = 6 \times 10^{18} \text{ cm}^{-3}$, respectively. The electron energy spectrum peak is ~ 200 MeV, with a higher energy tail. The electron beam itself was composed of 2 distinct bunches, with energies ~ 300 -350 MeV and < 215 MeV, respectively. Although the higher energy bunch is predicted to dominate the radiation pattern, the low energy bunch produced far better agreement with the experimental beam profile. This may be explained by the fact that the lower energy bunch had $\sim 10\times$ more charge in it than the high energy bunch. The simulation parameters, listed below Figure 3, were informed by the experimental data and adjusted in order to obtain a best fit with the experimental beam profile. The vertical direction of the simulation displays a nearly precise agreement. The lack of agreement in the horizontal direction may be due to the finite transverse width of the electron beam or contributions from lower energy electrons and different electron trajectories.

Betatron profiles with nearly circular shapes (i.e. $\theta^{(h)} \simeq \theta^{(v)}$) were better modeled by multiple electrons, injected along a distribution of angles. Varying the number of electrons and the angular orientation enabled beam profiles of shapes ranging from circular to elliptical to be successfully modeled.

IV. DISCUSSION

The results of these simulations are intended to be viewed as approximations to the true betatron

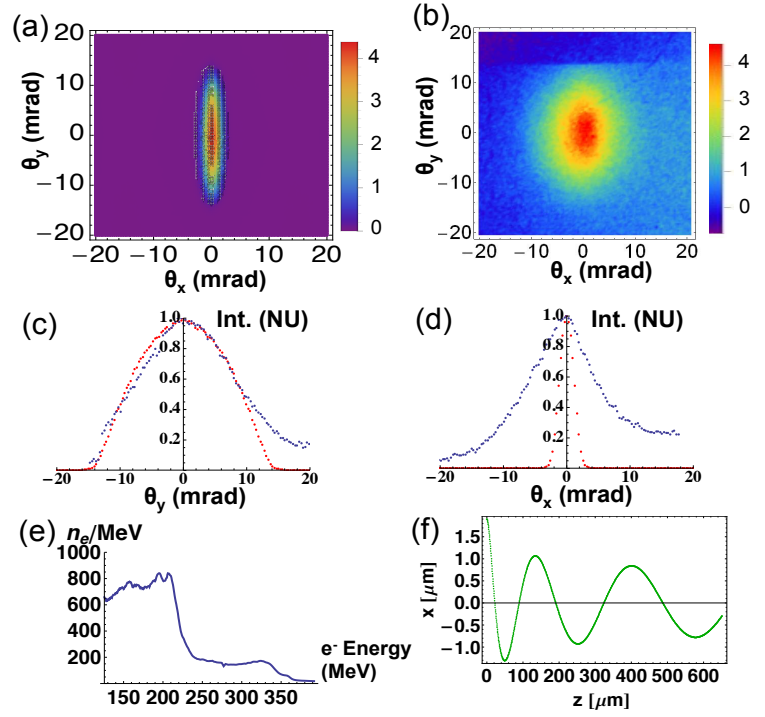


FIG 2 (color). (a) Simulated betatron profile. (b) Experimental profile. (c) Vertical slices of (a) (red) and (b) (blue), normalized. (d) Horizontal slices of (a) (red) and (b) (blue), normalized. (e) Experimental electron energy spectrum. (f) Simulated electron trajectory (right). Simulation parameters were: $\gamma_{\text{Final}} = 423$ (~ 215 MeV), $r_{0\beta} = 1.8 \mu\text{m}$, $\Delta\omega = 25$ eV, $\Delta_{x,y} = 0.32$ mrad ($\sim 200 \mu\text{m}$ at $z = 62.35$ cm).

motion. More precise results could be obtained if greater computing power was available. This would enable us to treat the number and orientation of electrons, $r_{0\beta}$, and γ_{Final} as free parameters over a reasonable range and to apply an automated optimization to obtain the closest agreement with experiment. Simulations will continue to be run on a wider variety of experimental results so that potential correlations between experimental parameters and betatron characteristics may be obtained.

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